

نهايات الدوال العددية  
1<sup>ère</sup> STE + 1<sup>ère</sup> STM

**التمرين 1 :** احسب النهايات التالية :

1.  $\lim_{x \rightarrow -\infty} (x^4 + 3x^2 - 2x)$  ؛  $\lim_{x \rightarrow +\infty} (3x^4 + 3x^2 + x)$  ؛  $\lim_{x \rightarrow -\infty} (-2x^3 + x^2 - 2x - 2)$  ؛  $\lim_{x \rightarrow +\infty} (-2x^3 + x^2 - 2x - 2)$  ( 1

2.  $\lim_{x \rightarrow 6} \frac{x^3 + x^2 - 12}{x + 6}$  ؛  $\lim_{x \rightarrow \sqrt{3}} (\sqrt{x^2 + 1} - x^2)$  ؛  $\lim_{x \rightarrow 0} \frac{-3x^2 + x - 1}{1 + 2x^3}$  ؛  $\lim_{x \rightarrow -2} (x^4 + 3x^2 - 2x)$  ؛  $\lim_{x \rightarrow 1} (x^2 - 5x + 3)$  ( 2

3.  $\lim_{x \rightarrow -\infty} \frac{4x^2 + 2x + 3}{x^3 + 2x + 4}$  ؛  $\lim_{x \rightarrow +\infty} \frac{-3x^2 + x - 1}{1 + 2x^3}$  ؛  $\lim_{x \rightarrow -\infty} \frac{-2x^2 + 2x + 5}{x^2 + 2x - 1}$  ؛  $\lim_{x \rightarrow +\infty} \frac{-3x^3 + x - 1}{1 - 2x^3}$  ( 3

4.  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{3}{x^2} \right)$  ؛  $\lim_{x \rightarrow 1} \frac{x^3 + x^2 + x - 5}{x - 1}$  ؛  $\lim_{x \rightarrow 1} \frac{2x^2 - 2x + 3}{(x - 1)^2}$  ؛  $\lim_{x \rightarrow 2} \frac{2x^2 - 3x - 1}{x - 2}$  ( 4

5.  $\lim_{x \rightarrow 1} \frac{3x^2 - 5x + 2}{x^2 + x - 2}$  ؛  $\lim_{x \rightarrow 2} \frac{x^3 + x^2 - 12}{x - 2}$  ؛  $\lim_{x \rightarrow -5} \frac{x^2 + 3x - 10}{x + 5}$  ؛  $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 6x + 8}$  ؛  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$  ( 5

6.  $\lim_{x \rightarrow 1} \frac{\sqrt{x + 3} - 2}{x - 1}$  ؛  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - x^2)$  ؛  $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1} - x}{x + 3}$  ؛  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1} + x}{x}$  ( 6

**التمرين 2 :**

احسب نهايات الدالة f عند محددات مجموعة تعريفها في كل حالة من الحالات التالية :

$$f(x) = \frac{x^3 - 2x^2 - 2x - 3}{-2x^2 - 8x + 42} \quad (6)$$

$$f(x) = \frac{(2+x)^3 + 8}{x} \quad (7)$$

$$f(x) = \frac{x^n - 3x + 1}{x^3 - 1} \quad (8)$$

$$f(x) = \frac{1}{x - 2} - \frac{4x^2}{x^3 - 8} \quad (9)$$

$$f(x) = 3x^5 - 5x^2 - 14 \quad (1)$$

$$f(x) = x - 4x^4 + 1 \quad (2)$$

$$f(x) = \frac{3x^2 + 2x - 1}{x + 1} \quad (3)$$

$$f(x) = \frac{6x^3 + 5x^2 - x - 1}{2x^2 + 9x + 4} \quad (4)$$

$$f(x) = \frac{3x + |x|}{3x - 2|x|} \quad (5)$$

**التمرين 3 :**

احسب النهايات التالية :

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\sin x \cos x} \quad (7)$$

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cos x - \sqrt{3} \sin x}{x - \frac{\pi}{6}} \quad (8)$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{2 \cos x - \sqrt{2}} \quad (8)$$

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{\tan x (\tan(x - \frac{\pi}{3}))}{1 - 2 \cos x} \quad (4)$$

$$\lim_{x \rightarrow 0} \frac{\sin x (\cos 2x - \cos x)}{\tan x - \sin x} \quad (5)$$

$$\lim_{x \rightarrow 1} \frac{\sin 3\pi x - 3 \sin \pi x}{x - 1} \quad (6)$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x} \quad (1)$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x}} \quad (2)$$

$$\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos 3x} \quad (3)$$

**التمرين 4 :**

1) احسب النهايات التالية :  $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x + \sqrt{3} \cos x}{x + \frac{\pi}{3}}$  ؛  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sqrt{1 - \cos x}}$  ؛  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

2.  $\lim_{x \rightarrow 2} \frac{\sqrt{x + 2} - 2}{\sqrt{x + 7} - 3}$  ؛  $\lim_{x \rightarrow 0} \frac{1}{x^2} \left( \frac{2}{\cos x} + \cos x - 3 \right)$  ؛  $\lim_{x \rightarrow 0} \frac{\sqrt{x + 1} - 1}{\tan x}$  ؛  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - 2 \cos x}{\sqrt{2} - 2 \sin x}$

2) نعتبر الدالة f المعرفة بما يلي :  $f(x) = \frac{4 \sin x + 3x}{x - 1}$

أ - بين أن :  $\forall x \in ]1; +\infty[ ; |f(x) - 3| \leq \frac{7}{x - 1}$  ؛ ب - استنتج :  $\lim_{x \rightarrow +\infty} f(x)$